A Spin 3/2 20-dimensional Wave Equation with Definite Charge and its Propagation in an External Electromagnetic Field

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Z. Naturforsch. **41a**, 1024–1028 (1986); received March 10, 1986

The charge of a spin 3/2, 20-dimensional wave equation of the Gel'fand-Yaglom form based on the representation (1/2, 3/2) ⊕ (− 1/2, 3/2) ⊕ (1/2, 5/2) ⊕ (− 1/2, 5/2) ⊕ (1/2, 3/2) ⊕ (− 1/2, 3/2) and its propagation behaviour in an external electromagnetic field are studied, and it is shown that its charge is definite but its propagation is not causal.

1. Introduction

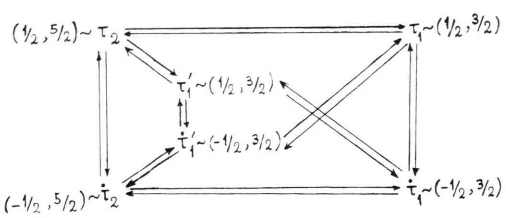
In this paper we shall be concerned with an example of a wave equation of the Gel'fand-Yoglom form [1, 2, 3]

$$\mathbb{L}_0 \frac{\partial \psi}{\partial x_0} + \mathbb{L}_1 \frac{\partial \psi}{\partial x_1} + \mathbb{L}_2 \frac{\partial \psi}{\partial x_2} + \mathbb{L}_3 \frac{\partial \psi}{\partial x_3} + i\chi\psi = 0 \qquad (1)$$

based on the representation

$$[\tau_1 \sim (1/2, 3/2)] \oplus [\dot{\tau}_1 \sim (-1/2, 3/2)] \oplus [\tau_2 \sim (1/2, 5/2)] \oplus [\dot{\tau}_2 \sim (-1/2, 5/2)] \oplus [\tau_1' \sim (1/2, 3/2)] \oplus [\dot{\tau}_1' \sim (-1/2, 3/2)] \qquad (2)$$

with components τ interlocking according to the scheme



(3)

(Our notation is the same as that of [3].)

The example we shall be concerned with is a 20-dimensional wave equation and describes particles of spin 3/2 with definite charge. We shall also look upon its propagation behaviour in an external electromagnetic field.

The canonical form of the wave equation (1) (in the general case) with respect to the canonical basis [3]

$$\{\xi_{lm}\} = \{\xi^{\tau_1}_{l_1, m_1}, \xi^{\dot{\tau}_1}_{l_1, m_1}, \xi^{\dot{\tau}_2}_{l_1, m_1}, \xi^{\dot{\tau}_2}_{l_2, m_2}, \xi^{\tau_2}_{l_1, m_1}, \xi^{\tau_2}_{l_2, m_2}, \xi^{\tau_1'}_{l_1, m_1}, \xi^{\dot{\tau}_1'}_{l_1, m_1}\}, \qquad (4)$$

$$l_1 = 1/2, \quad m_1 = 1/2, -1/2, \quad l_2 = 3/2, \quad m_2 = 3/2, 1/2, -1/2, -3/2,$$

which is invariant under the complete group, derivable from an invariant Lagrangian and associated with the bilinear form (ψ_1, ψ_2) defined by the constants [4]

$$\alpha^{\tau_1 \dot{\tau}_1} = \alpha^{\dot{\tau}_1 \tau_1} = 1, \quad \alpha^{\tau_2 \dot{\tau}_2} = \alpha^{\dot{\tau}_2 \tau_2} = -1, \quad \alpha^{\tau_1' \dot{\tau}_1'} = \alpha^{\dot{\tau}_1' \tau_1} = 1 \qquad (5)$$

has a matrix $\mathbb{L}_0$ which in block form reads:

for $l = 1/2$,

$$\mathbb{L}_0(l = 1/2) = \begin{array}{c|cccccc} & \tau_1 & \dot{\tau}_1 & \dot{\tau}_2 & \tau_2 & \tau_1' & \dot{\tau}_1' \\ \hline \tau_1 & 0 & \alpha & 0 & i\sqrt{3}\,\beta & 0 & \gamma \\ \dot{\tau}_1 & \alpha & 0 & i\sqrt{3}\,\beta & 0 & \gamma & 0 \\ \dot{\tau}_2 & 0 & i\sqrt{3}\,\bar{\beta} & 0 & \varepsilon & 0 & i\sqrt{3}\,\zeta \\ \tau_2 & i\sqrt{3}\,\bar{\beta} & 0 & \varepsilon & 0 & i\sqrt{3}\,\zeta & 0 \\ \tau_1' & 0 & \bar{\gamma} & 0 & i\sqrt{3}\,\bar{\zeta} & 0 & \theta \\ \dot{\tau}_1' & \bar{\gamma} & 0 & i\sqrt{3}\,\bar{\zeta} & 0 & \theta & 0 \end{array}$$

and for $l = 3/2$,

$$\mathbb{L}_0(l = 3/2) = \begin{array}{c|cc} & \dot{\tau}_2 & \tau_2 \\ \hline \dot{\tau}_2 & 0 & 2\varepsilon \\ \tau_2 & 2\varepsilon & 0 \end{array},$$

where $\alpha, \beta, \gamma, \varepsilon, \zeta, \theta$ are constants and $i = \sqrt{-1}$. The constants $\alpha, \varepsilon, \theta$ are real numbers. For simplicity we

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introduce the new constants $b = i\sqrt{3}\,\beta$, $c = i\sqrt{3}\,\bar{\beta}$, $z = i\sqrt{3}\,\zeta$, $k = i\sqrt{3}\,\bar{\zeta}$. ((−) indicates the complex conjugate.) Restricting our selves to equations describing spin 3/2 particles with or without spin 1/2 particles present (depending on the eigenvalues of the block $\mathbb{L}_0(l = 1/2)$) we can devide throughout by 2ε, in which case the block $\mathbb{L}_0(l = 3/2)$ aquires nonvanishing eigenvalues. Renaming the constants in this case as

$$C = \frac{c}{2\varepsilon}, \quad Z = \frac{z}{2\varepsilon}, \quad B = \frac{b}{2\varepsilon}, \quad A = \frac{\alpha}{2\varepsilon},$$

$$\Gamma = \frac{\gamma}{2\varepsilon}, \quad K = \frac{k}{2\varepsilon}, \quad \Theta = \frac{\theta}{2\varepsilon}, \tag{6}$$

the blocks of $\mathbb{L}_0$ read:

for $l = 1/2$

$$\mathbb{L}_0(l = 1/2) = \begin{array}{c} \\ \tau_1 \\ \dot{\tau}_1 \\ \dot{\tau}_2 \\ \tau_2 \\ \tau_1' \\ \dot{\tau}_1' \end{array} \left(\begin{array}{cccccc} \tau_1 & \dot{\tau}_1 & \dot{\tau}_2 & \tau_2 & \tau_1' & \dot{\tau}_1' \\ 0 & A & 0 & B & 0 & \Gamma \\ A & 0 & B & 0 & \Gamma & 0 \\ 0 & C & 0 & \frac{1}{2} & 0 & Z \\ C & 0 & \frac{1}{2} & 0 & Z & 0 \\ 0 & \bar{\Gamma} & 0 & K & 0 & \Theta \\ \bar{\Gamma} & 0 & K & 0 & \Theta & 0 \end{array} \right)$$

and for $l = 3/2$ (7)

$$\mathbb{L}_0(l = 3/2) = \begin{array}{c} \\ \dot{\tau}_2 \\ \tau_2 \end{array} \begin{array}{c} \begin{array}{cc} \dot{\tau}_2 & \tau_2 \end{array} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}.$$

2. A Special Example and Charge Associated with it

If the constants A, B, Γ, C, Z, K, Θ in the above wave equation have the values

$$B = \frac{1}{2\sqrt{2}}, \quad C = -\frac{1}{2\sqrt{2}}, \quad Z = -\frac{1}{2\sqrt{2}}, \quad K = \frac{1}{2\sqrt{2}},$$

$$A = -\frac{1}{4}, \quad \Gamma = -\frac{1}{4}, \quad \Theta = -\frac{1}{4}, \tag{8}$$

a wave equation of the Gel'fand-Yaglom form results which describes particles of spin 3/2. The matrix $\mathbb{L}_0$ associated with this wave equation has the following blocks for spins 1/2 and 3/2, respectively:

for $l = 1/2$

$$\mathbb{L}_0(l = 1/2) = \begin{array}{c} \\ \tau_1 \\ \dot{\tau}_1 \\ \dot{\tau}_2 \\ \tau_2 \\ \tau_1' \\ \dot{\tau}_1' \end{array} \begin{array}{c} \begin{array}{cccccc} \tau_1 & \dot{\tau}_1 & \dot{\tau}_2 & \tau_2 & \tau_1' & \dot{\tau}_1' \end{array} \\ \begin{pmatrix} 0 & -\frac{1}{4} & 0 & \frac{1}{2\sqrt{2}} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{2\sqrt{2}} & 0 & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} & 0 & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2} & 0 & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & -\frac{1}{4} & 0 & \frac{1}{2\sqrt{2}} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{2\sqrt{2}} & 0 & -\frac{1}{4} & 0 \end{pmatrix} \end{array}$$

and for $l = 3/2$ (9)

$$\mathbb{L}_0(l = 3/2) = \begin{array}{c} \\ \dot{\tau}_2 \\ \tau_2 \end{array} \begin{array}{c} \begin{array}{cc} \dot{\tau}_2 & \tau_2 \end{array} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}.$$

We notice that the eigenvalues of the block $\mathbb{L}_0(l = 1/2)$ are all zero and the equation describes only spin 3/2 particles. The matrix $\mathbb{L}_0$ satisfies the minimal equation

$$\mathbb{L}_0^2 \{\mathbb{L}_0^2 - 1\} = 0. \tag{10}$$

Thus $\mathbb{L}_0$ is not a diagonalizable matrix.

We look next upon the charge associated with the above wave equation defined by the formula

$$q = \int \psi^+ \mathbb{A} \mathbb{L}_0 \psi \, \mathrm{d}v, \tag{11}$$

where $\mathbb{A}$ is the hermitianizing matrix associated with the representation (2) and can be expressed in terms of the constants $\alpha^{\dot{\tau}_i \tau_j}$ of the bilinear form given in (5). ψ^+ is the complex conjugate transpose of ψ.

Using $\mathbb{L}_0$ given by (9) and $\mathbb{A}$ we find that the charge associated with the wave equation considered here is definite.

3. Spinor Formulation of the Wave Equation

We reformulate the above wave equation with matrix $\mathbb{L}_0$ given by (9) in spinor language. This we do because it is much easier using the spinor form of the wave equation to find the subsidiary conditions of the second kind. These conditions are necessary in the study of the propagation of the wave equation by means of the method of characteristics.

To express the wave equation (1) in spinor form it is necessary to find the similarity transformation $\mathbb{T}$ connecting the canonical basis (4) to the spinor basis [5, 6]

$$\{a_{\varphi v}^{\dot\omega}, d^{\dot\omega}, \delta^{\dot\omega}, b_v^{\dot\omega\dot\varphi}, c_\omega, \gamma_\omega\}, \tag{12}$$

where $\dot\omega = \dot 1, \dot 2$, $\varphi = 1, 2$, $v = 1, 2$, $\dot\varphi = \dot 1, \dot 2$, $\omega = 1, 2$. The transformation $\mathbb{T}$ must be such that when acting on the generators and matrices of the wave equation in the canonical basis it converts them into the corresponding generators and matrices of the wave equation in the spinor basis, namely

$$\mathbb{T} H_3^c \mathbb{T}^{-1} = \mathbb{H}_3^s, \quad \mathbb{T}\mathbb{H}_\pm^c \mathbb{T}^{-1} = \mathbb{H}_\pm^s, \quad \mathbb{T}\mathbb{F}_3^c \mathbb{T}^{-1} = \mathbb{F}_3^s,$$

$$\mathbb{T}\mathbb{F}_\pm^c, \mathbb{T}^{-1}\mathbb{F}_\pm^s, \mathbb{T}\mathbb{L}_j^s \mathbb{T}^{-1} = \mathbb{L}_j^s, \tag{13}$$

where c and s stand for the canonical and spinor frames, respectively, and $j = 0, 1, 2, 3$. These relations are sufficient to determine $\mathbb{T}$. Using this $\mathbb{T}$ we find the spinorial form of the general 20-dimensional Gel'fand-Yaglom wave equation for maximum spin 3/2 and matrix $\mathbb{L}_0$ given by (7). In the particular case where the constants A, B, Γ, C, Z, K, Θ have the values given by (8), the corresponding wave equation in the spinor basis reads

$$-\pi_{1\dot 1} b_1^{\dot 1\dot 1} - \pi_{1\dot 2} b_1^{\dot 1\dot 2} - \frac{1}{\sqrt2}\pi_1^{\dot 1} c_1 - \frac{1}{\sqrt2}\pi_1^{\dot 1}\gamma_1 + \chi a_{11}^{\dot 1} = 0, \tag{E1}$$

$$-\frac12 \pi_{2\dot 2} b_1^{\dot 1\dot 2} - \frac12 \pi_{1\dot 1} b_2^{\dot 1\dot 1} - \frac{1}{2\sqrt2}\pi_2^{\dot 1} c_1 - \frac{1}{2\sqrt2}\pi_2^{\dot 1}\gamma_1 - \frac12 \pi_{1 2} b_1^{\dot 1\dot 1} - \frac12 \pi_{1\dot 2} b_2^{\dot 1\dot 2} - \frac{1}{2\sqrt2}\pi_1^{\dot 1} c_2 - \frac{1}{2\sqrt2}\pi_1^{\dot 1}\gamma_2 + \chi a_{12}^{\dot 1} = 0, \tag{E2}$$

$$-\pi_{2\dot 2} b_2^{\dot 1\dot 2} - \frac{1}{\sqrt2}\pi_2^{\dot 1} c_2 - \frac{1}{\sqrt2}\pi_2^{\dot 1}\gamma_2 - \pi_{2\dot 1} b_2^{\dot 1\dot 1} + \chi a_{22}^{\dot 1} = 0, \tag{E3}$$

$$-\pi_{1\dot 1} b_1^{\dot 1\dot 2} - \frac{1}{\sqrt2}\pi_1^{\dot 2} c_1 - \frac{1}{\sqrt2}\pi_1^{\dot 2}\gamma_1 - \pi_{1\dot 2} b_1^{\dot 2\dot 2} + \chi a_{11}^{\dot 2} = 0, \tag{E4}$$

$$-\frac12 \pi_{\dot 2 2} b_1^{\dot 2\dot 2} - \frac12 \pi_{1\dot 1} b_2^{\dot 1\dot 2} - \frac{1}{2\sqrt2}\pi_1^{\dot 2} c_2 - \frac{1}{2\sqrt2}\pi_1^{\dot 2}\gamma_2 - \frac12 \pi_{\dot 2 1} b_1^{\dot 1\dot 2} - \frac12 \pi_{1\dot 2} b_2^{\dot 2\dot 2} - \frac{1}{2\sqrt2}\pi_2^{\dot 2} c_1 - \frac{1}{2\sqrt2}\pi_2^{\dot 2}\gamma_1 + \chi a_{12}^{\dot 2} = 0, \tag{E5}$$

$$-\pi_{2\dot 2} b_2^{\dot 2\dot 2} - \pi_{2\dot 1} b_2^{\dot 1\dot 2} - \frac{1}{\sqrt2}\pi_2^{\dot 2} c_2 - \frac{1}{\sqrt2}\pi_2^{\dot 2}\gamma_2 + \chi a_{22}^{\dot 2} = 0, \tag{E6}$$

$$-\frac{1}{6\sqrt2}\pi_{\dot 2}^{1} b_1^{\dot 1\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 1}^{2} b_2^{\dot 1\dot 1} + \frac14 \pi^{1\dot 1} c_1 + \frac14 \pi^{1\dot 1}\gamma_1 - \frac{1}{6\sqrt2}\pi_{\dot 1}^{1} b_1^{\dot 1\dot 1} - \frac{1}{6\sqrt2}\pi_{\dot 2}^{2} b_2^{\dot 1\dot 2} + \frac14 \pi^{\dot 1 2} c_2 + \frac14 \pi^{\dot 1 2}\gamma_2 + \chi d^{\dot 1} = 0, \tag{E7}$$

$$-\frac{1}{6\sqrt2}\pi_{\dot 2}^{1} b_1^{\dot 2\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 1}^{2} b_2^{\dot 1\dot 2} + \frac14 \pi^{\dot 2 2} c_2 + \frac14 \pi^{\dot 2 2}\gamma_2 - \frac{1}{6\sqrt2}\pi_{\dot 1}^{1} b_1^{\dot 1\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 2}^{2} b_2^{\dot 2\dot 2} + \frac14 \pi^{\dot 2 1} c_1 + \frac14 \pi^{\dot 2 1}\gamma_1 + \chi d^{\dot 2} = 0, \tag{E8}$$

$$-\frac{1}{6\sqrt2}\pi_{\dot 2}^{1} b_1^{\dot 1\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 1}^{2} b_2^{\dot 1\dot 1} + \frac14 \pi^{1\dot 1} c_1 + \frac14 \pi^{1\dot 1}\gamma_1 - \frac{1}{6\sqrt2}\pi_{\dot 1}^{1} b_2^{\dot 1\dot 1} - \frac{1}{6\sqrt2}\pi_{\dot 2}^{2} b_2^{\dot 1\dot 2} + \frac14 \pi^{\dot 1 2} c_2 + \frac14 \pi^{\dot 1 2}\gamma_2 + \chi \delta^{\dot 1} = 0, \tag{E9}$$

$$-\frac{1}{6\sqrt2}\pi_{\dot 2}^{1} b_1^{\dot 2\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 1}^{2} b_2^{\dot 1\dot 2} + \frac14 \pi^{\dot 2 2} c_2 + \frac14 \pi^{\dot 2 2}\gamma_2 - \frac{1}{6\sqrt2}\pi_{\dot 1}^{1} b_1^{\dot 1\dot 2} - \frac{1}{6\sqrt2}\pi_{\dot 2}^{2} b_2^{\dot 2\dot 2} + \frac14 \pi^{\dot 2 1} c_1 + \frac14 \pi^{\dot 2 1}\gamma_1 + \chi \delta^{\dot 2} = 0, \tag{E10}$$

$$-\pi^{\dot 1 1} a_{11}^{\dot 1} - \pi^{\dot 1 2} \alpha_{12}^{\dot 1} - \frac{1}{\sqrt2}\pi_{\dot 1}^{1} d^{\dot 1} - \frac{1}{\sqrt2}\pi_{\dot 1}^{1}\delta^{\dot 1} + \chi b_1^{\dot 1\dot 1} = 0, \tag{E11}$$

$$-\frac12 \pi^{\dot 2 2} a_{12}^{\dot 1} - \frac12 \pi^{\dot 1 1} a_{11}^{\dot 2} - \frac{1}{2\sqrt2}\pi_{\dot 1}^{\dot 2} d^{\dot 1} - \frac{1}{2\sqrt2}\pi_{\dot 1}^{\dot 2}\delta^{\dot 1} - \frac12 \pi^{\dot 2 1} a_{11}^{\dot 1} - \frac12 \pi^{\dot 1 2} a_{12}^{\dot 2} - \frac{1}{2\sqrt2}\pi_{\dot 1}^{1} d^{\dot 2} - \frac{1}{2\sqrt2}\pi_{\dot 1}^{1}\delta^{\dot 2} + \chi b_1^{\dot 1\dot 2} = 0, \tag{E12}$$

$$-\pi^{\dot 2 2} a_{12}^{\dot 2} - \frac{1}{\sqrt2}\pi_{\dot 1}^{\dot 2} d^{\dot 2} - \frac{1}{\sqrt2}\pi_{\dot 1}^{\dot 2}\delta^{\dot 2} - \pi^{\dot 2 1} a_{11}^{\dot 2} + \chi b_1^{\dot 2\dot 2} = 0, \tag{E13}$$

$$-\pi^{\dot 1 1} a_{12}^{\dot 1} - \frac{1}{\sqrt2}\pi_{\dot 2}^{\dot 1} d^{\dot 1} - \frac{1}{\sqrt2}\pi_{\dot 2}^{\dot 1}\delta^{\dot 1} - \pi^{\dot 1 2} a_{22}^{\dot 1} + \chi b_2^{\dot 1\dot 1} = 0, \tag{E14}$$

$$-\frac{1}{2}\pi^{\dot{2}2}a^{\dot{1}}_{22}-\frac{1}{2}\pi^{\dot{1}1}a^{\dot{2}}_{12}-\frac{1}{2\sqrt{2}}\pi^{\dot{1}}_{2}d^{\dot{2}}-\frac{1}{2\sqrt{2}}\pi^{\dot{1}}_{2}\delta^{\dot{2}}$$
$$-\frac{1}{2}\pi^{\dot{2}1}a^{\dot{1}}_{12}-\frac{1}{2}\pi^{\dot{1}2}a^{\dot{2}}_{22}-\frac{1}{2\sqrt{2}}\pi^{\dot{2}}_{2}d^{\dot{1}}$$
$$-\frac{1}{2\sqrt{2}}\pi^{\dot{2}}_{2}\delta^{\dot{1}}+\chi b^{\dot{1}\dot{2}}_{2}=0, \qquad (E\,15)$$

$$-\pi^{\dot{2}2}a^{\dot{2}}_{22}-\pi^{\dot{2}1}a^{\dot{2}}_{12}-\frac{1}{\sqrt{2}}\pi^{\dot{2}}_{2}d^{\dot{2}}-\frac{1}{\sqrt{2}}\pi^{\dot{2}}_{2}\delta^{\dot{2}}+\chi b^{\dot{2}\dot{2}}_{2}=0, \qquad (E\,16)$$

$$-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{1}}a^{\dot{1}}_{12}-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{2}}a^{\dot{2}}_{11}+\frac{1}{4}\pi_{\dot{1}1}d^{\dot{1}}+\frac{1}{4}\pi_{\dot{1}1}\delta^{\dot{1}}$$
$$-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{1}}a^{\dot{1}}_{11}-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{2}}a^{\dot{2}}_{12}+\frac{1}{4}\pi_{1\dot{2}}d^{\dot{2}}$$
$$+\frac{1}{4}\pi_{1\dot{2}}\delta^{\dot{2}}+\chi c_{1}=0, \qquad (E\,17)$$

$$-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{1}}a^{\dot{1}}_{22}-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{2}}a^{\dot{2}}_{12}+\frac{1}{4}\pi_{\dot{2}2}d^{\dot{2}}+\frac{1}{4}\pi_{\dot{2}2}\delta^{\dot{2}}$$
$$-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{1}}a^{\dot{1}}_{12}-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{2}}a^{\dot{2}}_{22}+\frac{1}{4}\pi_{2\dot{1}}d^{\dot{1}}$$
$$+\frac{1}{4}\pi_{2\dot{1}}\delta^{\dot{1}}+\chi c_{2}=0, \qquad (E\,18)$$

$$-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{1}}a^{\dot{1}}_{22}-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{2}}a^{\dot{2}}_{11}+\frac{1}{4}\pi_{\dot{1}1}d^{\dot{1}}+\frac{1}{4}\pi_{\dot{1}1}\delta^{\dot{1}}$$
$$-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{1}}a^{\dot{1}}_{11}-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{2}}a^{\dot{2}}_{12}+\frac{1}{4}\pi_{1\dot{2}}d^{\dot{2}}$$
$$+\frac{1}{4}\pi_{1\dot{2}}\delta^{\dot{2}}+\chi \gamma_{1}=0, \qquad (E\,19)$$

$$-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{1}}a^{\dot{1}}_{22}-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{2}}a^{\dot{2}}_{12}+\frac{1}{4}\pi_{\dot{2}2}d^{\dot{2}}+\frac{1}{4}\pi_{\dot{2}2}\delta^{\dot{2}}$$
$$-\frac{1}{6\sqrt{2}}\pi^{1}_{\dot{1}}a^{\dot{1}}_{12}-\frac{1}{6\sqrt{2}}\pi^{2}_{\dot{2}}a^{\dot{2}}_{22}+\frac{1}{4}\pi_{2\dot{1}}d^{\dot{1}}$$
$$+\frac{1}{4}\pi_{2\dot{1}}\delta^{\dot{1}}+\chi \gamma_{2}=0, \qquad (E\,20)$$

4. Subsidiary Conditions

As was mentioned in the previous paragraph, to be able to study the propagation of the wave equation in the presence of an external field using the method of characteristics it is necessary to find the subsidiary conditions of the second kind which are determined from the above spinorial form of the wave equation. Thus let us multiply (E 11) by $\sqrt{2}\,\pi^{1}_{\dot{1}}$, (E 12) by $\sqrt{2}\,\pi^{1}_{\dot{2}}$, (E 14) by $\sqrt{2}\,\pi^{2}_{\dot{1}}$ and (E 15) by $\sqrt{2}\,\pi^{2}_{\dot{2}}$ and add, i.e.

$$\sqrt{2}\,\pi^{1}_{\dot{1}}\times(E_{11})+\sqrt{2}\,\pi^{1}_{\dot{2}}\times(E_{12})+\sqrt{2}\,\pi^{2}_{\dot{1}}\times(E_{14})+\sqrt{2}\,\pi^{2}_{\dot{2}}\times(E_{15})=0. \qquad (14)$$

Similarly let us multiply (E 17) by $3\pi^{\dot{1}1}$, (E 18) by $3\pi^{2\dot{1}}$, (E 19) by $3\pi^{1\dot{1}}$ and (E 20) by $3\pi^{2\dot{1}}$ and add, i.e.

$$3\pi^{1\dot{1}}\times(E_{17})+3\pi^{2\dot{1}}\times(E_{18})+3\pi^{1\dot{1}}\times(E_{19})+3\pi^{2\dot{1}}\times(E_{20})=0. \qquad (15)$$

Subtracting (15) from (14) we have

$$\sqrt{2}\{\pi^{1}_{\dot{1}}\times(E_{11})+\pi^{1}_{\dot{2}}\times(E_{12})+\pi^{2}_{\dot{1}}\times(E_{14})+\pi^{2}_{\dot{2}}\times(E_{15})\}$$
$$-3\{\pi^{1\dot{1}}\times(E_{17})+\pi^{2\dot{1}}\times(E_{18})+\pi^{\dot{1}1}\times(E_{19})+\pi^{2\dot{1}}\times(E_{20})\}=0. \qquad (16)$$

Substituting into this the relation

$$-6\chi\times(E_{7})-6\chi\times(E_{9}) \qquad (17)$$

(having imposed the conditions

$$B+K=\frac{1}{\sqrt{2}},\quad A+\bar{\Gamma}=-\tfrac{1}{2},\quad \Gamma+\Theta=-\tfrac{1}{2} \qquad (18)$$

in the general spinorial form of the wave equation), replacing $\pi_{\sigma\dot{\varrho}}$ by the relations

$$\pi_{1\dot{1}}=-\pi_{0}+\pi_{3},\quad \pi_{2\dot{1}}=\pi_{1}+i\,\pi_{2},$$
$$\pi_{1\dot{2}}=\pi_{1}-i\,\pi_{2},\quad \pi_{2\dot{2}}=-\pi_{0}-\pi_{3} \qquad (19)$$

and substituting $[\pi_{p},\pi_{q}]=ieF_{pq}=f_{pq}$ (where F_{pq}, $p,q=0,1,2,3$, is the electromagnetic tensor) (and finally imposing the conditions

$$3\sqrt{2}\,C-3(A+\bar{\Gamma})=0,\quad 3\sqrt{2}\,Z-3(\Gamma+\Theta)=0 \qquad (20)$$

in the general spinorial form of the wave equation, in order to make terms involving $(\pi_r)^2$, $r=0, 1, 2, 3\ldots$ vanish) we obtain the following subsidiary condition of the second kind:

$$6\chi^{2}d^{\dot{1}}+6\chi^{2}\delta^{\dot{1}}+\sqrt{2}(f_{10}+f_{13}+if_{32}+if_{02})\,a^{\dot{1}}_{11}$$
$$+2\sqrt{2}(f_{03}+if_{12})\,a^{\dot{1}}_{12}-2(if_{12}+f_{30})\,d^{\dot{1}}$$
$$-2(f_{30}+if_{12})\,\delta^{\dot{1}}-2(f_{10}+if_{20}+f_{13}+if_{23})\,d^{\dot{2}}$$
$$-2(f_{10}+f_{13}+if_{20}+if_{23})\,\delta^{\dot{2}}$$
$$+\sqrt{2}(f_{01}+if_{02}+if_{23}+f_{13})\,a^{\dot{1}}_{22}=0. \qquad (21)$$

By similar operations we obtain the following three extra subsidiary conditions:

$$\begin{aligned} &6\chi^2 d^{\dot 2} + 6\chi^2 \delta^{\dot 2} + \sqrt{2}(f_{10} + f_{13} + i f_{02} + i f_{32})\, a^{\dot 2}_{11} \\ &- 2(f_{10} + i f_{02} + f_{31} + i f_{23})\, d^{\dot 1} \\ &- 2(f_{10} + i f_{02} + f_{31} + i f_{32})\, \delta^{\dot 1} + 2\sqrt{2}(i f_{12} + f_{03})\, a^{\dot 2}_{12} \\ &- 2(i f_{21} + f_{03})\, d^{\dot 2} - 2(i f_{21} + f_{03})\, \delta^{\dot 2} \\ &+ \sqrt{2}(f_{01} + i f_{02} + f_{13} + i f_{23})\, a^{\dot 2}_{22} = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} &6\chi^2 c_1 + 6\chi^2 \gamma_1 + \sqrt{2}(f_{10} + f_{31} + i f_{02} + i f_{23})\, b^{\dot 1 \dot 1}_{1} \\ &+ 2\sqrt{2}(i f_{21} + f_{03})\, b^{\dot 1 \dot 2}_{1} - 2(i f_{12} + f_{03})\, c_1 \\ &- 2(i f_{12} + f_{03})\, \gamma_1 - 2(f_{01} + i f_{02} + i f_{23} + f_{13})\, c_2 \\ &- 2(f_{01} + i f_{02} + f_{13} + i f_{23})\, \gamma_2 \\ &+ \sqrt{2}(f_{01} + i f_{02} + f_{31} + i f_{32})\, b^{\dot 2 \dot 2}_{1} = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} &6\chi^2 c_2 + 6\chi^2 \gamma_2 + \sqrt{2}(f_{10} + f_{31} + i f_{02} + i f_{23})\, b^{\dot 1 \dot 1}_{2} \\ &- 2(f_{01} + i f_{20} + f_{31} + i f_{23})\, c_1 \\ &- 2(f_{01} + f_{31} + i f_{20} + i f_{23})\, \gamma_1 + 2\sqrt{2}(f_{03} + i f_{21})\, b^{\dot 1 \dot 2}_{2} \\ &- 2(i f_{21} + f_{30})\, c_2 - 2(i f_{21} + f_{30})\, \gamma_2 \\ &+ \sqrt{2}(f_{01} + i f_{02} + f_{31} + i f_{32})\, b^{\dot 2 \dot 2}_{2} = 0. \end{aligned} \tag{24}$$

Besides the above four subsidiary conditions of the second kind involving the external field components f_{kl} the wave equation (E) accepts also four more subsidiary conditions of the second kind involving no external field components. Thus let us consider (E 17) and (E 19). Subtracting (E 19) from (E 17) we find the secondary constraint

$$c_1 = \gamma_1. \tag{25}$$

Subtracting (E 20) from (E 18) we find

$$c_2 = \gamma_2. \tag{26}$$

Also, subtracting (E 9) from (E 7) we find

$$d^{\dot 1} = \delta^{\dot 1}. \tag{27}$$

Finally subtracting (E 10) from (E 8) we find

$$d^{\dot 2} = \delta^{\dot 2}. \tag{28}$$

5. Propagation in an External Electromagnetic Field

We look now upon the propagation of the above wave equation in the presence of an external electromagnetic field. Thus let us consider the constraints (25), (26), (27), and (28) and substitute them into the wave equation. Doing so we find that (E 20) is the same as (E 18), (E 19) is the same as (E 17), (E 10) is the same as (E 8) and (E 9) is the same as (E 7). Thus it can be dispenced with (E 20), (E 19), (E 10), and (E 9) and reduce the problem to one involving sixteen differential equations. Furthermore, changing the spinor basis

$$\{a^{\dot 1}_{11}, a^{\dot 1}_{12}, a^{\dot 1}_{22}, a^{\dot 2}_{11}, a^{\dot 2}_{12}, a^{\dot 2}_{22}, d^{\dot 1}, d^{\dot 2}, b^{\dot 1 \dot 1}_{1}, b^{\dot 1 \dot 2}_{1}, b^{\dot 2 \dot 2}_{1}, b^{\dot 1 \dot 1}_{2}, b^{\dot 1 \dot 2}_{2}, b^{\dot 2 \dot 2}_{2}, c_1, c_2\} \tag{29}$$

to the new basis

$$\{a^{\dot 1}_{11}, a^{\dot 1}_{12}, a^{\dot 1}_{22}, a^{\dot 2}_{11}, a^{\dot 2}_{12}, a^{\dot 2}_{22}, \sqrt{2}, d^{\dot 1}, \sqrt{2}, d^{\dot 2}, b^{\dot 1 \dot 1}_{1}, b^{\dot 1 \dot 2}_{1}, b^{\dot 2 \dot 2}_{1}, b^{\dot 1 \dot 1}_{2}, b^{\dot 1 \dot 2}_{2}, b^{\dot 2 \dot 2}_{2}, \sqrt{2}\, c_1, \sqrt{2}\, c_2\} \tag{30}$$

obtained from the old one by scaling up the spin 1/2 components by the factor $\sqrt{2}$, the set of sixteen differential equations with respect to the new basis becomes the same as the Pauli-Fierz wave equation while the subsidiary conditions of the second kind involving the field f_{kl} with respect to new basis become the same as those of the Pauli-Fierz wave equation. Hence we conclude that the propagation of the spin 3/2 wave equation considered in this paper is the same as that of the Pauli-Fierz wave equation studied by the author in detail in [7] and shown to violate causality in the presence of an external electromagnetic field.

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